

# Output Feedback Based Admissible Control of Switched Linear Singular Systems<sup>1)</sup>

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**Abstract** The admissibility analysis and robust admissible control problem of the uncertain discrete-time switched linear singular (SLS) systems for arbitrary switching laws are investigated. Based on linear matrix inequalities, some sufficient conditions are given for: A) the existence of generalized common Lyapunov solution and the admissibility of the SLS systems for arbitrary switching laws, B) the existence of static output feedback control laws ensuring the admissibility of the closed-loop SLS systems for arbitrary switching laws and norm-bounded uncertainties.

**Key words** Switched system, singular system, admissibility, static output feedback

## 1 Introduction

Recently, the problem of stability and stabilization of switched systems has attracted considerable attention in the area of systems and control<sup>[1,2]</sup>. Among others, a series of methods and conditions based on common quadratic Lyapunov functions (CQLFs) have been given for analyzing the stability of switched systems for arbitrary switching laws<sup>[1,2]</sup>. It was pointed out in [3] that the problem of finding a CQLF was one of the unsolved problems in mathematical systems and control theory. For discrete-time systems which have only two stable second order discrete time linear subsystems, a necessary and sufficient condition on the existence of CQLF was given in [2]. But for the general cases of higher order or more subsystems, it seems that there is still no a necessary and sufficient condition on the existence of CQLF.

Switched linear singular (SLS) system is an important class of switched systems, which arises from, for example, electrical networks and economic systems<sup>[4~6]</sup>. Due to the existence of switching actions, state-inconsistence phenomena often occur in the electrical networks. This may result in discontinuity of network variables and presence of impulse voltage and currents at the switching instants. For dynamic economic systems, as pointed out by Cantó *et al.* in [5], when the interrelationships among different industrial sectors are described, and the capital and the demand are variables depending on seasons, the system can be modelled as periodically switched singular systems. Both analysis and synthesis of switched singular systems are more difficult, since stability, regularity, impulse elimination and state consistence of such systems should be considered at the same time.

In this paper, we will analyze the admissibility (*i.e.*, the regularity, causality and stability) of discrete-time SLS systems for arbitrary switching laws and norm-bounded uncertainties. The key difference from the conventional switched systems is that the dimensions of the dynamic parts of each subsystem of SLS systems may be different from each other. So, the conventional Lyapunov frame is not suitable for the stability analysis of SLS systems. It is pointed out that for the SLS systems with a generalized common Lyapunov function the dynamic parts of subsystems have the same dimension. Furthermore, the conditions of the existence of generalized common Lyapunov solution and static output feedback control laws ensuring the admissibility of the closed-loop SLS systems for arbitrary switching laws and norm-bounded uncertainties are presented based on LMIs.

## 2 Notations and preliminary results

Consider the following uncertain discrete-time switched linear singular (SLS) system:

$$\begin{cases} E_{\sigma}\mathbf{x}(k+1) = (A_{\sigma} + \Delta A_{\sigma})\mathbf{x}(k) + B_{\sigma}\mathbf{u}_{\sigma}(k) \\ \mathbf{y}_{\sigma}(k) = C_{\sigma}\mathbf{x}(k) \end{cases} \quad (1)$$

1) Supported by National Natural Science Foundation of P. R. China (6022130, 60334040, 60428304)  
Received May 13, 2005; in revised form October 17, 2005

where  $\sigma : \{0, 1, \dots\} \rightarrow \Lambda = \{1, 2, \dots, m\}$  is the switching law;  $\mathbf{x}(k) \in \mathbb{R}^n$ ,  $\mathbf{u}_i(k) \in \mathbb{R}^{m_i}$ ,  $\mathbf{y}_i(k) \in \mathbb{R}^{p_i}$ ,  $i \in \Lambda$ , are the state, input and output, respectively;  $E_i \in \mathbb{R}^{n \times n}$ ,  $\text{rank} E_i = n_i < n$ ;  $A_i \in \mathbb{R}^{n \times n}$ ,  $B_i \in \mathbb{R}^{n \times m_i}$ ,  $C_i \in \mathbb{R}^{p_i \times n}$ ,  $i \in \Lambda$ ;  $\Delta A_i$ ,  $i \in \Lambda$ , is the norm-bounded parameter uncertainty of the form

$$\Delta A_i = M_i F_i(\rho) N_i \quad (2)$$

Here  $M_i \in \mathbb{R}^{n \times s_i}$ ,  $N_i \in \mathbb{R}^{t_i \times n}$  are known real constant matrices, and the uncertain matrix  $F_i(\rho) \in \mathbb{R}^{s_i \times t_i}$  satisfies

$$F_i(\rho) F_i(\rho)^T \leq I, \quad \forall \rho \in \Sigma \text{ with } \Sigma \text{ being a compact set} \quad (3)$$

**Remark 1.**  $\Delta A_i$ ,  $i \in \Lambda$ , is the so-called ‘‘norm-bounded uncertainties’’ and is considered frequently in the robust control.

Throughout this paper,  $\mathbb{C}$  denotes the set of all complex numbers,  $\mathbb{R}^n$  denotes the real  $n$ -dimensional space;  $\mathbb{R}^{n \times n}$  denotes the real  $n \times n$ -dimensional space; for a given vector or matrix  $X$ ,  $X^T$  denotes its transpose,  $\text{rank} X$  denotes its rank,  $X^{(1)}$  denotes a 1-inverse of  $X$ ,  $\mathcal{R}(X)$  denotes the subspace spanned by the columns of  $X$ ;  $\mathcal{N}(X)$  denotes the right zero subspace of  $X$ ; for an  $n \times m$  full column rank matrix  $A$ ,  $A^\perp$  denotes an  $n \times (n - m)$  matrix with the following properties:  $A^{\perp T} A = 0$ ,  $[A \ A^\perp]$  is invertible and  $A^{\perp T} A^\perp = I$ ; and for a square matrix  $X$ ,  $\lambda(X)$  denotes its spectrum.

**Definition 1.**<sup>[7]</sup> Consider two SLS systems:

$$\Sigma_1 : \begin{cases} E_\sigma \mathbf{x}(k+1) = A_\sigma \mathbf{x}(k) + B_\sigma \mathbf{u}(k) \\ \mathbf{y}(k) = C_\sigma \mathbf{x}(k) \end{cases} \quad \text{and} \quad \Sigma_2 : \begin{cases} \bar{E}_\sigma \bar{\mathbf{x}}(k+1) = \bar{A}_\sigma \bar{\mathbf{x}}(k) + \bar{B}_\sigma \mathbf{u}(k) \\ \mathbf{y}(k) = \bar{C}_\sigma \bar{\mathbf{x}}(k) \end{cases}$$

If there exist nonsingular matrices  $Q_i$  and  $P$  such that  $Q_i E_i P = \bar{E}_i$ ,  $Q_i A_i P = \bar{A}_i$ ,  $Q_i B_i = \bar{B}_i$ ,  $C_i P = \bar{C}_i$ ,  $i \in \Lambda$ , and  $\bar{\mathbf{x}} = P^{-1} \mathbf{x}$ , then systems  $\Sigma_1$  and  $\Sigma_2$  are called restricted system equivalent (r.s.e.).

**Remark 2.** It should be pointed out that in Definition 1, the transformation matrix  $P$  is uniform with respect to  $i \in \Lambda$ . This is helpful in finding one state space coordinate basis for all the subsystems so that the stability analysis and feedback control can conveniently and concisely be done via the same coordinate basis.

**Definition 2.** Consider the SLDS system

$$E_\sigma \mathbf{x}(k+1) = A_\sigma \mathbf{x}(k) \quad (4)$$

1) For a given  $i \in \Lambda$ , the pair  $(E_i, A_i)$  is said to be regular if there exists a constant scalar  $s_i \in \mathbb{C}$  such that  $\det(s_i E_i - A_i) \neq 0$ .

The SLS system (4) is said to be regular if every pair  $(E_i, A_i)$ ,  $i \in \Lambda$ , is regular.

2) For a given  $i \in \Lambda$ , the pair  $(E_i, A_i)$  is said to be causal if it is regular and  $\deg(\det(s_i E_i - A_i)) = \text{rank} E_i$  for all  $s_i \in \mathbb{C}$ .

The SLS system (4) is said to be casual if every pair  $(E_i, A_i)$ ,  $i \in \Lambda$ , is casual.

**Definition 3.** The SLS system (4) is said to be admissible for arbitrary switching law if it is regular, causal and asymptotically stable for arbitrary switching laws.

**Remark 3.** By Definitions 1-3 one can see that the admissibility of a singular system is preserved under restricted system equivalent transformation.

**Lemma 1**<sup>[8]</sup>. Given matrices  $\Omega$ ,  $\Gamma$  and  $\Xi$  of appropriate dimensions and with  $\Omega$  symmetrical,

$$\Omega + \Gamma F(\rho) \Xi + (\Gamma F(\rho) \Xi)^T < 0 \quad \text{for all } F(\rho) \text{ satisfying } F(\rho)^T F(\rho) \leq I$$

if and only if there exists a scalar  $\epsilon > 0$  such that  $\Omega + \epsilon \Gamma \Gamma^T + \epsilon^{-1} \Xi^T \Xi < 0$ .

### 3 Generalized common lyapunov solution

Similar to [1], we will briefly call the symmetric matrix  $P$  satisfying

$$E_i P E_i^T \geq 0, \quad A_i P A_i^T - E_i P E_i^T < 0, \quad \forall i \in \Lambda \quad (5)$$

a generalized common Lyapunov solution (GCLS).

In this section, under a certain condition, we give a strict LMI-based condition for the GCLS existence of SLS systems, and show that if an SLS system has a GCLS, then it is admissible for arbitrary switching laws.

**Proposition 1.** If there exists a GCLS for system (4), then for all  $i \in \Lambda$ ,  $E_i$  has the same rank.

**Proof.** From the definition of the GCLS, there exists a symmetric matrix  $P$  such that (5) holds. This implies that for any fixed  $i$ ,  $(E_i, A_i)$  is admissible<sup>[9]</sup>. Thus, there exist matrices  $M_i$  and  $N_i$ ,  $i \in \Lambda$ , such that<sup>[7]</sup>  $M_i E_i N_i = \text{diag}[I_{n_i} \ 0]$ ,  $M_i A_i N_i = \text{diag}[G_i \ I_{n-n_i}]$ . Denote  $N_i^{-1} P N_i^{-T} = \begin{bmatrix} P_{i11} & P_{i12} \\ P_{i12}^T & P_{i22} \end{bmatrix}$ , where  $P_{i11} \in \mathbb{R}^{n_i \times n_i}$  and  $P_{i22} \in \mathbb{R}^{(n-n_i) \times (n-n_i)}$  are symmetric,  $P_{i12} \in \mathbb{R}^{n_i \times (n-n_i)}$ . By the proof of Lemma 1 of [9], we know  $P_{i11} > 0$ ,  $P_{i11} - P_{i12} P_{i22}^{-1} P_{i12}^T > 0$ , and  $P_{i22} < 0$ ,  $i \in \Lambda$ .

Let

$$R_i = \begin{bmatrix} (P_{i11} - P_{i12} P_{i22}^{-1} P_{i12}^T)^{-\frac{1}{2}} & 0 \\ 0 & P_{i22}^{-\frac{1}{2}} \end{bmatrix} \begin{bmatrix} I_{n_i} & -P_{i12} P_{i22}^{-1} \\ 0 & I_{n-n_i} \end{bmatrix}, \quad i \in \Lambda$$

Then  $R_i N_i^{-1} P N_i^{-T} R_i^T = \text{diag}[I_{n_i} \ -I_{n-n_i}]$ ,  $i \in \Lambda$ . This together with the inertia law of quadratic forms<sup>[10]</sup> leads to  $n_1 = \dots = n_m$ . Thus, for all  $i \in \Lambda$ ,  $E_i$  has the same rank.  $\square$

In the sequel, we assume that for all  $i \in \Lambda$ ,  $E_i$  has the same rank, and denote the rank by  $r$ . In order to analyze the admissibility of system (4), we need to find the transformation matrices  $M_i$  and  $N$ ,  $i \in \Lambda$ , such that system (4) is r.s.e. to

$$\begin{cases} \mathbf{x}_1(k+1) = A_{\sigma 11} \mathbf{x}_1(k) + A_{\sigma 12} \mathbf{x}_2(k) \\ A_{\sigma 21} \mathbf{x}_1(k) + A_{\sigma 22} \mathbf{x}_2(k) = 0 \end{cases} \quad (6)$$

where  $M_i E_i N = \text{diag}[I_r \ 0]$ ,  $M_i A_i N = \begin{bmatrix} A_{i11} & A_{i12} \\ A_{i21} & A_{i22} \end{bmatrix}$ ,  $N^{-1} \mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix}$ ,  $A_{i11} \in \mathbb{R}^{r \times r}$ ,  $\mathbf{x}_1 \in \mathbb{R}^r$ .

**Lemma 2.** There exist transformation matrices  $M_i$ ,  $i \in \Lambda$ , and  $N$  such that system (4) is r.s.e. to (6) if and only if  $\mathcal{N}(E_1) = \dots = \mathcal{N}(E_m)$ .

**Proof.** For necessity, suppose that there exist transformation matrices  $M_i$  and  $N$  such that system (4) is r.s.e. to (6) and let  $N = [\bar{N} \ \Phi]$ . Then, it is easy to see that  $\mathcal{R}(\Phi) = \mathcal{N}(E_i)$ ,  $i \in \Lambda$ . Hence,  $\mathcal{N}(E_1) = \dots = \mathcal{N}(E_m)$ .

For sufficiency, suppose that  $\mathcal{N}(E_1) = \dots = \mathcal{N}(E_m)$ . Then, one can choose  $\Phi \in \mathbb{R}^{n \times (n-r)}$  such that for any  $i \in \Lambda$ ,  $E_i \Phi = 0$ ; and, there are a matrix  $\bar{N}$  and nonsingular matrix  $M_i \in \mathbb{R}^{n \times n}$ ,  $i \in \Lambda$ , such that  $N = [\bar{N} \ \Phi]$  nonsingular and  $M_i E_i N = \text{diag}[I_r \ 0]$ . Thus, system (4) is r.s.e. to (6).  $\square$

So, in the following discussion we assume:

**Assumption 1.** For any  $i \in \Lambda$ ,  $\mathcal{N}(E_i)$  is the same.

**Theorem 1.** Under Assumption 1, the discrete-time SLS system (4) is admissible for arbitrary switching laws if there exists a symmetric matrix  $P \in \mathbb{R}^{n \times n}$  such that (5) holds.

**Proof.** By the proof of Lemma 2, we know that under Assumption 1, one can find matrices  $M_i \in \mathbb{R}^{n \times n}$ ,  $i \in \Lambda$ , and  $N \in \mathbb{R}^{n \times n}$  such that system (4) is r.s.e. to (6). By Remark 3, the admissibility of (4) and (6) is equivalent. So, we need only to prove that (6) is admissible. Suppose that there exists a symmetric matrix  $P \in \mathbb{R}^{n \times n}$  such that (5) holds. Then from [9] we know that the subsystems  $(E_i, A_i)$ ,  $i \in \Lambda$ , are causal and regular. So, by [7],  $A_{i22}$ ,  $i \in \Lambda$ , are nonsingular. Furthermore, by Definition 2, the switched system (4) is causal and regular. Let  $N^{-1} P N^{-T} = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix}$ . Then by (5), we have  $P_{11} > 0$  and  $\bar{A}_{i1} P_{11} \bar{A}_{i1}^T - P_{11} < 0$  where  $\bar{A}_{i1} = A_{i11} - A_{i12} A_{i22}^{-1} A_{i21}$ . This implies that  $I - P_{11}^{-\frac{1}{2}} \bar{A}_{i1} P_{11} \bar{A}_{i1}^T P_{11}^{-\frac{1}{2}} > 0$ . Then it follows that  $0 < \alpha \leq 1$  where  $\alpha = \min_{i \in \Lambda} \lambda(I - P_{11}^{-\frac{1}{2}} \bar{A}_{i1} P_{11} \bar{A}_{i1}^T P_{11}^{-\frac{1}{2}})$ . By (6) and the non-singularity of  $A_{i22}$ , we have

$$\mathbf{x}_2(k) = -A_{\sigma 22}^{-1} A_{\sigma 21} \mathbf{x}_1(k) \quad (7)$$

and

$$\mathbf{x}_1(k+1) P_{11} \mathbf{x}_1^T(k+1) = \mathbf{x}_1(k) P_{11} \mathbf{x}_1^T(k) - \mathbf{x}_1(k) (P_{11} - \bar{A}_{\sigma 1} P_{11} \bar{A}_{\sigma 1}^T) \mathbf{x}_1^T(k) \leq (1 - \alpha)^{k+1} \mathbf{x}_1(0) P_{11} \mathbf{x}_1^T(0)$$

This implies that the sub-state  $\mathbf{x}_1(k)$  is asymptotically convergent to zero, and so is the  $\mathbf{x}_2(k)$  by (7). Therefore, system (4) is admissible for arbitrary switching laws.  $\square$

Theorem 1 gives a sufficient condition under which system (4) is admissible for arbitrary switching laws. The following corollary transforms the matrix inequality conditions to strict LMIs conditions.

**Corollary 1.** Under Assumption 1, there exists a symmetric matrix solution  $P$  such that (5) holds if and only if there exist symmetric matrices  $X \in \mathbb{R}^{n \times n}$  and  $Y \in \mathbb{R}^{(n-r) \times (n-r)}$ ,  $X > 0$ , such that

$$A_i(X + \Phi Y \Phi^T)A_i^T - E_i X E_i^T < 0, \quad \forall i \in \Lambda \quad (8)$$

**Proof.** For sufficiency, suppose that there exist matrices  $X \in \mathbb{R}^{n \times n}$ ,  $Y \in \mathbb{R}^{(n-r) \times (n-r)}$ , and  $X > 0$  such that (8) holds. Let  $P = X + \Phi Y \Phi^T$ . Then, by  $E_i \Phi = 0$  we can see  $E_i P E_i^T = E_i X E_i^T \geq 0$ ,  $A_i P A_i^T - E_i P E_i^T = A_i(X + \Phi Y \Phi^T)A_i^T - E_i X E_i^T < 0$ . Thus, (5) holds.

For necessity, suppose  $P$  is a symmetric matrix solution of (5). Choose a symmetric matrix  $X_{22} \in \mathbb{R}^{(n-r) \times (n-r)}$  such that

$$\begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & X_{22} \end{bmatrix} > 0 \quad (9)$$

This is feasible since from the proof procedure of Theorem 1 we have  $P_{11} > 0$ , and by Schur Complements<sup>[11]</sup> it suffices to choose  $X_{22} > P_{12}^T P_{11}^{-1} P_{12}$ .

Let

$$X = N \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & X_{22} \end{bmatrix} N^T \quad \text{and} \quad Y = P_{22} - X_{22} \quad (10)$$

Then, by (9), the first equation of (10) and the nonsingularity of  $N$  we have  $X > 0$ . And by  $\Phi = N[0 \ I_{n-r}]^T$  and (9) we have  $P = X + \Phi Y \Phi^T$ . Thus, (8) follows from the second inequality of (5).  $\square$

From Theorem 1 and Corollary 1 we can easily have

**Corollary 2.** Under Assumption 1, if there exist matrices  $X \in \mathbb{R}^{n \times n}$ ,  $X > 0$ , and a symmetric matrix  $Y \in \mathbb{R}^{(n-r) \times (n-r)}$ , such that  $A_i(X + \Phi^T Y \Phi)A_i^T - E_i X E_i^T < 0$ ,  $\forall i \in \Lambda$ , then the discrete-time SLS system (4) is admissible for arbitrary switching laws.

#### 4 Robust admissibility of SLS systems

Consider the following uncertain discrete-time SLS system:

$$E_\sigma \mathbf{x}(k+1) = (A_\sigma + \Delta A_\sigma) \mathbf{x}(k) \quad (11)$$

where  $E_i$ ,  $\sigma$ ,  $\mathbf{x}(k)$ ,  $A_i$ , and  $\Delta A_i$ ,  $i \in \Lambda$ , are as in (1).

The purpose of this section is to give a sufficient condition in terms of strict LMIs for the admissibility of uncertain discrete-time SLS system (11) for arbitrary switching laws. Let

$$P(X, Y) = X + \Phi Y \Phi^T \quad (12)$$

By a similar proof procedure to those of Theorem 1 and Corollary 1, we have

**Lemma 3.** Under Assumption 1, if there exist symmetric matrices  $X \in \mathbb{R}^{n \times n}$  and  $Y \in \mathbb{R}^{(n-r) \times (n-r)}$ ,  $X > 0$ , such that

$$(A_i + \Delta A_i)P(X, Y)(A_i + \Delta A_i)^T - E_i X E_i^T < 0, \quad \forall i \in \Lambda \quad (13)$$

then the uncertain discrete-time SLS system (11) is admissible for arbitrary switching laws, where  $P(X, Y)$  is given by (12).

**Theorem 2.** Under Assumption 1, if there exist symmetric matrices  $X \in \mathbb{R}^{n \times n}$  and  $Y \in \mathbb{R}^{(n-r) \times (n-r)}$ ,  $X > 0$ , and scalars  $\epsilon_i > 0$ ,  $\gamma_i > 0$ ,  $i \in \Lambda$ , such that

$$\begin{bmatrix} A_i P(X, Y) A_i^T - E_i X E_i^T + \gamma_i M_i M_i^T & A_i P(X, Y) N_i^T \\ (A_i P(X, Y) N_i^T)^T & -(\gamma_i I - Q_i) \end{bmatrix} < 0, \quad i \in \Lambda \quad (14)$$

then the uncertain SLS system (11) is admissible for arbitrary switching laws, where  $Q_i = \epsilon_i I + N_i P(X, Y) N_i^T$ , and  $P(X, Y)$  is given by (12).

#### 5 Output feedback based robust admissible control

The purpose of this section is to design a robust static output feedback control for the SLS system (1) to ensure the admissibility of the closed-loop system for arbitrary switching laws. It was pointed out that the problem of finding the static output feedback matrix is NP-hard<sup>[12]</sup>. To overcome this difficulty, we give a matrix inequality condition for the existence of the feedback matrix, which can be solved easily by using the MatLab LMI ToolBox.

The output feedback controller to be used is of the form:

$$\mathbf{u}_\sigma(k) = G_\sigma \mathbf{y}_\sigma(k) \quad (15)$$

where  $G_i \in \mathbb{R}^{m_i \times p_i}$ ,  $i \in \Lambda$ . And the corresponding closed-loop system is of the following form:

$$E_\sigma \mathbf{x}(k+1) = (\bar{A}_\sigma + \Delta A_\sigma) \mathbf{x}(k) \quad (16)$$

where  $\bar{A}_\sigma = A_\sigma + B_\sigma G_\sigma C_\sigma$ .

**Theorem 3.** Under Assumption 1, if there exist matrices  $X \in \mathbb{R}^{n \times n}$ ,  $X > 0$ ,  $S_i \in \mathbb{R}^{n \times n}$ ,  $T_i \in \mathbb{R}^{n \times n}$ ,  $L_i \in \mathbb{R}^{m_i \times p_i}$ , nonsingular matrices  $F_i \in \mathbb{R}^{p_i \times p_i}$ , a symmetric matrix  $Y \in \mathbb{R}^{(n-r) \times (n-r)}$ , scalars  $\epsilon_i > 0$ , and  $\gamma_i > 0$ , such that  $\forall i \in \Lambda$ ,

$$\begin{bmatrix} A_{i1} & A_i T_i - B_i L_i C_i - S_i^T & A_i P(X, Y) N_i^T + B_i L_i C_i N_i^T \\ (A_i T_i - B_i L_i C_i - S_i^T)^T & P(X, Y) - T_i - T_i^T & 0 \\ (A_i P(X, Y) N_i^T + B_i L_i C_i N_i^T)^T & 0 & -(\gamma_i I - Q_i) \end{bmatrix} < 0 \quad (17)$$

$$F_i C_i = C_i T_i, \quad F_i C_i = C_i S_i, \quad F_i C_i = C_i P(X, Y) \quad (18)$$

where  $A_{i1} = A_i S_i + S_i^T A_i^T - B_i L_i C_i - C_i^T L_i^T B_i^T - E_i X E_i^T + \gamma_i M_i M_i^T$ ,  $Q_i = N_i P(X, Y) N_i^T + \epsilon_i I$ ,  $P(X, Y)$  is given by (12), then the closed-loop system (16) is admissible for arbitrary switching laws provided that the feedback matrix is chosen as

$$G_i = -L_i F_i^{-1}, \quad i \in \Lambda \quad (19)$$

In the case where  $\Delta A_i = 0$ ,  $\forall i \in \Lambda$ , the closed-loop system is of the following form:

$$E_\sigma \mathbf{x}(k+1) = \bar{A}_\sigma \mathbf{x}(k) \quad (20)$$

where  $\bar{A}_\sigma = A_\sigma + B_\sigma G_\sigma C_\sigma$ . By Theorem 3, we have

**Corollary 3.** Under Assumption 1, if there exist matrices  $X \in \mathbb{R}^{n \times n}$ ,  $X > 0$ ,  $S_i \in \mathbb{R}^{n \times n}$ ,  $T_i \in \mathbb{R}^{n \times n}$ ,  $L_i \in \mathbb{R}^{m_i \times p_i}$ , symmetric matrix  $Y \in \mathbb{R}^{(n-r) \times (n-r)}$ , and nonsingular matrix  $F_i \in \mathbb{R}^{p_i \times p_i}$ , such that  $\forall i \in \Lambda$ ,

$$\begin{bmatrix} B_{i1} & A_i T_i - B_i L_i C_i - S_i^T \\ (A_i T_i - B_i L_i C_i - S_i^T)^T & P(X, Y) - T_i - T_i^T \end{bmatrix} < 0 \quad (21)$$

$$F_i C_i = C_i T_i, \quad F_i C_i = C_i S_i \quad (22)$$

where  $B_{i1} = A_i S_i + S_i^T A_i^T - B_i L_i C_i - C_i^T L_i^T B_i^T - E_i X E_i^T$ ,  $P(X, Y)$  is given by (12), then the closed-loop system (20) is admissible for arbitrary switching laws provided that the feedback matrix is chosen as (19).

According to the matrix theory, if matrix  $C_i$ ,  $i \in \Lambda$ , has full row rank (without loss of generality, we can assume that  $C_i$ ,  $i \in \Lambda$ , is of full row rank), equation (22) has solutions  $T_i$  and  $S_i$  if and only if

$$T_i = C_i^{(1)} F_i C_i + C_i^{T \perp} Y_{i1}, \quad \text{with } Y_{i1} \in \mathbb{R}^{(n-p_i) \times n} \text{ being an arbitrary matrix} \quad (23)$$

$$S_i = C_i^{(1)} F_i C_i + C_i^{T \perp} Y_{i2}, \quad \text{with } Y_{i2} \in \mathbb{R}^{(n-p_i) \times n} \text{ being an arbitrary matrix} \quad (24)$$

Here the notations  $X^{(1)}$  and  $X^\perp$  of a matrix  $X$  are stated in Section 2.

Substituting (23) and (24) into (21), we obtain the strict LMI conditions on the solution of output feedback of system (1) when  $\Delta A_i = 0$ ,  $\forall i \in \Lambda$ .

**Theorem 4.** Under Assumption 1, if there exist matrices  $X \in \mathbb{R}^{n \times n}$ ,  $X > 0$ ,  $Y_{i1} \in \mathbb{R}^{n-p_i \times n}$ ,  $Y_{i2} \in \mathbb{R}^{n-p_i \times n}$ ,  $L_i \in \mathbb{R}^{m_i \times p_i}$ ,  $F_i \in \mathbb{R}^{p_i \times p_i}$ , and symmetric matrix  $Y \in \mathbb{R}^{(n-r) \times (n-r)}$ , such that

$$\begin{bmatrix} C_{i1} & A_i T_i - B_i L_i C_i - S_i^T \\ (A_i T_i - B_i L_i C_i - S_i^T)^T & P(X, Y) - T_i - T_i^T \end{bmatrix} < 0, \quad \forall i \in \Lambda \quad (25)$$

where  $C_{i1} = A_i S_i + S_i^T A_i^T - B_i L_i C_i - C_i^T L_i^T B_i^T - E_i X E_i^T$ , and  $P(X, Y)$ ,  $T_i$  and  $S_i$  are given by (12), (23) and (24), then the closed-loop system (20) is admissible for arbitrary switching laws provided that the feedback matrices are chosen as (19).

**Remark 4.** If  $F_i$  is singular for some  $i \in A$ , then one can use  $G_i = -L_i(F_i + \rho_i I)^{-1}$  as a feedback gain matrix, where  $\rho_i$  is a sufficiently small positive number such that  $F_i + \rho_i I$  is nonsingular.

## 6 Numerical example

Consider an SLS system of the form:

$$\begin{cases} E_\sigma \mathbf{x}(k+1) = A_\sigma \mathbf{x}(k) + B_\sigma \mathbf{u}_\sigma(k) \\ \mathbf{y}_\sigma(k) = C_\sigma \mathbf{x}(k) \end{cases} \quad (26)$$

$$\text{with } \sigma \in A = \{1, 2\}, E_1 = E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A_1 = \begin{bmatrix} 1.4 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & -1 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 3 & -2 \end{bmatrix}$$

$$C_1 = [2 \quad 1 \quad 3], A_2 = \begin{bmatrix} 1.4 & 7 & 5 \\ 2 & 1 & 4 \\ 1 & -1 & 4 \end{bmatrix}, B_2 = \begin{bmatrix} -1 & 4 \\ 4 & 2 \\ 3 & -2 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix} \quad (27)$$

It can be verified that  $(E_1, A_1)$  is irregular, noncausal and unstable, and  $(E_2, A_2)$  is unstable. By (25) we have

$$X = \begin{bmatrix} 2.139 & -0.313 & 0.433 \\ -0.313 & 0.313 & -0.433 \\ 0.433 & -0.433 & 1.444 \end{bmatrix}, Y = -6.266, F_1 = 1.489, F_2 = \begin{bmatrix} 1.153 & -1.183 \\ -1.544 & 2.33 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} 0.075 \\ -0.230 \end{bmatrix}, Y_{11} = \begin{bmatrix} 2.044 & -1.212 & 0.327 \\ 1.536 & 1.69 & -0.715 \end{bmatrix}, Y_{12} = \begin{bmatrix} -0.472 & 0.039 & -1.396 \\ -0.929 & -0.202 & 0.336 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} 0.112 & -0.797 \\ 0.487 & 0.123 \end{bmatrix}, Y_{21} = [0.268 \quad 0.886 \quad 0.541], Y_{22} = [-0.601 \quad 1.756 \quad 2.701]$$

From (12) and (19) we have  $G_1, G_2$  and  $P(X, Y)$ :

$$G_1 = \begin{bmatrix} -0.051 \\ 0.155 \end{bmatrix}, G_2 = \begin{bmatrix} 1.129 & 0.915 \\ -1.542 & -0.836 \end{bmatrix}, P(X, Y) = \begin{bmatrix} 2.139 & -0.313 & 0.433 \\ -0.313 & 0.313 & -0.433 \\ 0.433 & -0.433 & -4.822 \end{bmatrix}$$

Then, under the output feedback laws  $\mathbf{u}_1(k) = G_1 \mathbf{y}(k)$  and  $\mathbf{u}_2(k) = G_2 \mathbf{y}(k)$ , the closed-loop SLS system (26) and (27) is admissible for arbitrary switching laws.

## 7 Concluding remarks

In this paper, the admissibility of discrete-time SLS systems for arbitrary switching laws and norm-bounded uncertainties is investigated. By using the LMI method, some sufficient conditions are derived for the existence of GCLS to ensure the admissibility for arbitrary switching laws of the SLS systems. Based on the matrix inequality condition, output feedback based robust admissible control is designed for discrete-time SLS systems for arbitrary switching laws and norm-bounded uncertainties. All the LMIs conditions given here are related with the system matrices directly, which can be easily solved *via* efficient LMI optimization algorithms such as LMI Toolbox.

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